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Summary of research activity

During the second year of my PHD I have mainly worked on two projects: the first one concerned the development of an exact local mapping from clock-spin partition functions to fermionic counterparts, and led to the publication [2]. The latter concerned the study of finite size effects on topological phases of matter and led to the preprint [3], which is currently undergoing peer-review. In the following I will discuss with some more details the two projects, describing the underlying motivations and the results obtained.

- With respect to usual spin operators, clock-spin operators obey a generalized Clifford algebra, where on-site commutation of clock-spins acquires a phase factor given by $e^{i2\pi/n}$, where n defines the clock-spin operators order. This inherent property gives clock-spins a genuine relation to (non-Abelian) anyons with the same on-site commutation relations, which are known as *parafermions*. Just as clock-spin models present a generalization of the quantum Ising model, the corresponding parafermionic models can be interpreted as generalizations of the Kitaev chain, where the topological phase is characterized by unpaired parafermionic modes exponentially localized at the boundaries. Such modes are interesting both from a fundamental perspective and due to their potential applications in topological quantum computation. Despite being responsible for their extremely rich physics, the exotic algebra proper of clock-spin operators make clock-spin models extremely difficult to study. With the aim of addressing this problem, in [2] I worked on the development of an exact local mapping from clock-spin to fermionic partition functions. The mapping is inspired by Fedotov and Popov's theory for spin models, and is based on (locally) representing n-th order clock-spin operators in terms of functions of n distinct fermionic creation and destruction operators. The excess states due to the enlargement of the Hilbert space are then ruled out in the computation of the fermionic partition function by adding an imaginary potential to the mapped fermionic Hamiltonian, which thus becomes effectively non-Hermitian. The main advantage of recasting the problem in fermionic language is that, by giving access to Wick's theorem, it enables the use of well established numerical tools based on diagrammatic expansions in the study of clock-spin models. Furthermore, in one dimension it allows to use bosonization to inspect the low energy properties of clock-spin models.

- The Haldane model represented the first theoretical proposal for a quantum anomalous Hall phase and is now regarded as the most famous model for a Chern insulator. In the topological phase, under open boundary conditions, is characterized by a gapped bulk and robust chiral edge sates, whose energies connect the bulk valence and conduction bands. Crucially, the time reversal doubling of the Haldane model results in the Kane-Mele model which, describing spinful fermions on a honeycomb lattice with strong spin-orbit interaction, is the prototypical model of a time reversal protected topological phase. Motivated by the rich research landscape in nanostructured topological insulators and by the renewed interest in honeycomb-based topological models due to the discovery of new experimentally accessible platforms, in [3] I considered zigzag Haldane nanoribbons and assessed the effects of dimensional reduction on the topological phase of the Haldane model. I found that, for thin enough strips, there are multiple regions of the parameter space in which the chiral edge states gap out and, correspondingly, degenerate pairs of quasi zero-dimensional (0D) end-states appear whose energy lies inside the gap. Such regions however are intercalated, through topological quantum phase transitions, to phases without bound states. The system is thus described by a complex, width-dependent, reentrant quantum phase diagram, in which the quantum phase transitions are accompanied by jumps of π in the Zak phase. Moreover, I also proved that domain walls in the on-site staggered potential distribution can localize quasi 0D bound states. These bound states are described in the framework of Jackiw-Rebbi theory, implying that they retain a fractional charge of $\pm\frac{e}{2}$ (e the electron charge). Finally, they are found to be robust against the introduction of random on-site disorder.

List of publications

[1] S. Traverso, M. Sassetti, and N. T. Ziani, Role of the Edges in a Quasicrystalline Haldane Model, Phys. Rev. B **106**, 125428 (2022).

[2] S. Traverso, C. Fleckenstein, M. Sassetti, and N. T. Ziani, An Exact Local Mapping from Clock-Spins to Fermions, SciPost Phys. Core 6, 055 (2023).

[3] S. Traverso, M. Sassetti, and N. T. Ziani, Emerging Topological Bound States in Haldane Model Zigzag Nanoribbons, arXiv:2307.14771 (2023).

List of attended courses and exams given

- Energetics in the quantum regime - Attended
- Advanced Computational Physics - Attended
- Hands-on Crash Course on Theoretical Condensed Matter Physics - Attended

List of schools/conferences attended

- 108° *Congresso Nazionale SIF*, Milano (MI), Italia, 12/09/2022 - 16/09/2022. I attended the conference the day 15/09/2022 and I gave a presentation titled "Novel topological states in a thin strip of Haldane model".
- Workshop of Baltic Consortium of Theoretical Physics *Modern Aspects in Quantum Materials and Quantum Technology (MAQMQT22)*, Greifswald, Germany, 17/11/2022 - 20/11/2022. I presented a poster titled "Role of the edges is a quasicrystalline Haldane model".
- International Workshop *Korrelationstage 2023*, Dresden, Germany, 11/09/2023 - 15/09/2023. I presented a poster titled "The winding path from the Haldane model to Jackiw-Rebbi states".