Annual PhD report - Third year

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XXXVIII cycle

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1 Research activity

I am a PhD student in Theoretical Physics and this year I worked on applications of Quantum Field Theory to Condensed Matter, focusing in particular on the reloation between fractons and higher-rank gauge theories in $2+1$ dimensions. Fracton phases of matter constitute an interesting point of contact between Condensed Matter and High-Energy Physics. Fractons represent an extreme case of subdimensional quasiparticles, i.e. exotic excitations of certain quantum phases of matter which which are restricted to move only in lower dimensional subspaces. In particular, quasiparticles which can only move on zero-, one- and two-dimensional subspaces are known, respectively, as fractons, lineons and planons. The first realization of this subdimensional behavior has been made in exactly solvable quantum spin models with discrete symmetries as quantum error-correcting codes. The prototypical examples of fracton spin models in three spatial dimensions are the X-Cube and the Haah's Code which belong, respectively, to "type I" and "type II" classes, the former being characterized by all three types of subdimensional excitations while the latter has fractons only. Subsequently it was realized that the restricted mobility of subdimensional particles can be understood in terms of higher moment conservation laws which can be derived from symmetric $U(1)$ tensor gauge field theories. Relevantly, gapped fracton phases and gapless higher-rank symmetric $U(1)$ lattice gauge theories were related through the Higgsing procedure.

In the following I describe in further detail the two papers on which I worked this year.

- **Fractons from covariant higher-rank 3D BF theory [2]**
  In this paper we study the 3D gauge theory of two tensor gauge fields: $a_{\mu\nu}(x)$, which we take symmetric, and $B_{\mu\nu}(x)$, with no symmetry on its indices. The corresponding invariant action is a higher-rank BF-like model, which is first considered from a purely field theoretical point of view, and the propagators with their poles and the degrees of freedom are studied. Once matter is introduced, a fracton behaviour naturally emerges. We show that our theory can be mapped to the low-energy effective field theory describing the Rank-2 Toric Code (R2TC). This relation between our covariant BF-like theory and the R2TC is a higher-rank generalization of the equivalence between the ordinary 3D BF theory and the Kitaev's Toric Code. In the last part of the paper we analyze the case in which the field $B_{\mu\nu}(x)$ is a symmetric tensor. It turns out that the obtained BF-like action can be cast into the sum of two rank-2 Chern-Simons actions, thus generalizing the ordinary abelian case. Therefore, this represents a higher-rank generalization of the ordinary 3D BF theory, which well describes the low-energy physics of quantum spin Hall insulators in two spatial dimensions.

- **Covariant field theory of 3D massive fractons [1]**
  We construct a covariant and gauge-invariant theory describing massive fractons in three spacetime dimensions, based on a symmetric rank-2 tensor field. The model includes a Chern-Simons-like term that plays a dual role: it generates a topological mass for the tensor gauge field and simultaneously acts as a source of intrinsic fractonic matter. This dual mechanism is novel and leads to a propagating fractonic degree of freedom described by a massive Klein-Gordon equation. The theory propagates two degrees of freedom – one massive, one massless – whose number is preserved in the massless limit, in analogy with the Maxwell-Chern-Simons mechanism of Deser-Jackiw-Templeton. We analyze the resulting equations of motion and show that the intrinsic fractonic matter satisfies Gauss- and Ampère-like laws, with conserved dipole and trace of the quadrupole momentum. Upon coupling to external matter, a second

fractonic sector emerges, leading to a coexistence of intrinsic and extrinsic subsystems with different mobility and conservation properties. Our model provides a unified framework for describing massive fractons with internal structure, and offers a covariant setting for exploring their interactions and extensions.

2 Publications

[1] E. Bertolini, M. Carrega, N. Maggiore and D. Sacco Shaikh, *Covariant field theory of 3D massive fractons*. Submitted to *European Physical Journal C*.

[2] E. Bertolini, A. Blasi, M. Carrega, N. Maggiore and D. Sacco Shaikh, *Fractons from covariant higher-rank three-dimensional BF theory*, Phys. Rev. B 111, 085126 (2025).

[3] E. Bertolini, A. Blasi, N. Maggiore and D. Sacco Shaikh, *Hall-like behaviour of higher rank Chern-Simons theory of fractons*, J. High Energ. Phys. 2024, 232 (2024).

3 Courses and exams

I gave the following exams:

- *Introduction to AdS/CFT and its applications*, Dr. A. Amoretti (PhD course). I presented a seminar on *Magneto-transport from momentum dissipating holography*
- *Statistical Physics of Out of Equilibrium Systems*, Prof. N. Magnoli (Master Degree course). I presented a seminar on *Hydrodynamic fluctuations in relativistic theories*

4 Conferences and contributions

- *New Frontiers in Theoretical Physics - XXXVIII Convegno Nazionale di Fisica Teorica* (Cortona, Arezzo), 20-23 May 2025. I gave a talk: *Fractons from covariant higher-rank Chern-Simons and BF theories.*
- *StatPhys29, 29th International Conference on Statistical Physics* (Firenze), 13-18 July 2025. I presented a poster: *Fractons from covariant higher-rank three dimensional gauge theories.*
- *111° National Congress of the Italian Physical Society SIF 2022* (Palermo), 22-26 September 2025. I will give a talk: *Fractons from covariant higher-rank 3D gauge theories.*