Second Year PhD Report

Cycle: XXXIX

Student: Jonas Rongen

Supervisor: Prof. Andrea Amoretti

Research Activity

My research explores the intersection of hydrodynamics and holography. During the first year, my work focused primarily on hydrodynamics, while in the second year I shifted attention to holography, also known as gauge/gravity duality. Since gauge/gravity duality is a weak/strong coupling correspondence, it provides a powerful tool to study strongly interacting field theories by mapping their strong-coupling regime to the weak-coupling regime of gravity theories.

probe brane holography
One of the earliest applications of the AdS/CFT correspondence to condensed matter physics—particularly to strongly coupled quantum liquids—has been the study of holographic probe brane models at finite density. For the sake of completeness, the field-theoretical dual of these models is conjectured to be $\mathcal{N} = 4$ SU(N_c) supersymmetric Yang–Mills theory with $\mathcal{N} = 2$ supersymmetric fundamental matter. Much of the research in this area focuses on electrical transport of the conserved $U(1)$ charge.
By solving the linearised bulk equations of motion it has been shown that the current operator gives rise to a (quasi-)hydrodynamic description of a fluid subject to an external electric field E_i. This description includes a conservation equation for the charge density and a modified (non-)conservation equation for the charge current with a relaxation term. This means that at large charge density, the resulting effective hydrodynamic theory introduces the gapped pole into the relaxed hydrodynamic spectrum.

SL($2\mathbb{Z}$)
Building on this effective theory, my project was initially motivated by the question of whether analogous hydrodynamic descriptions could be obtained with large magnetic field instead of large charge density. This can be done by studying how the effective linearised theory transforms under SL($2\mathbb{Z}$) transformations.
As a first step, the simpler S-transformation on the effective theory has been considered, a subgroup of SL($2\mathbb{Z}$), which exchanges charge density with magnetic field. On the holographic side, where I considered the D3/D5 brane system, this corresponds to swapping the roles of source and vev in the boundary expansion, such that poles and zeros of the retarded Green's function exchange places. When the leading term in the boundary expansion is taken as the source and the subleading term as the vev, this is referred to as normal quantisation. Reversing the roles leads to alternate quantisation.
To obtain the transport coefficients of the effective theory I solved the equations of motion of the holographic model for the parallel and transverse part of the gauge field with respect to the wave vector in Eddington-Finkelstein coordinates with ingoing boundary conditions. The implemented shooting algorithm in normal quantisation allowed me to

numerically compute the retarded Green's function and identify its poles, also known as quasi-normal modes (QNMs). From these results, I was able to extract the transport coefficients of the non-transformed effective theory, thereby establishing the matching between the holographic model and its effective hydrodynamical description.
The effective theory we began with captures the quasihydrodynamic regime, in which a gapped pole lies very close to the hydrodynamic pole for large charge density. After applying an S-duality transformation to this effective theory, we again observed two poles: a hydrodynamic one and a gapped pole, which transitioned into quasihydrodynamic behaviour. However, when performing the corresponding alternate quantisation on the probe brane model, the spectrum differed: we recovered the hydrodynamic pole, but instead of the expected gapped pole we observed two poles that, with increasing density, moved away from the imaginary axis. This revealed that the effective hydrodynamical theory predicted a spurious pole absent in the correct holographic description. This mismatch led us to conclude that the original effective theory was incomplete, which in turn shifted the project towards a new research direction.

Mittag-Leffler expansion
To understand the emergence of quasihydrodynamics in the D3/D5 probe brane model, we examined the more general connection between the Mittag–Leffler expansion and holographic correlation functions of U(1) charge currents. Within a suitable disc around the origin of the complex frequency plane, a two-point function can be expressed through a Mittag–Leffler expansion, which rewrites the meromorphic function in terms of a holomorphic part and a sum over its poles. This representation provides the foundation for constructing a linearised effective theory that reproduces the expansion and correctly captures the conductivity behaviour. This general framework can then be applied to the D3/D5 probe brane system through a defined holographic approximant to the retarded Green's function.

Paper in Progress

- A. Amoretti, D. K. Brattan, J. Rongen, "Effective linearised theories for Mittag-Leffler expansions in holography"

Courses

I did the exams for the following courses

- Non-Abelian Gauge Theory (N. Maggiore, 3 CFU)
- Introduction to AdS/CFT (A. Amoretti, 3 CFU)

Conferences and Seminars

Talks

- Holographic perspectives on chiral transport and spin dynamics, Trento, Italy, 24-28 March 2025, Title: Dissipative electrically driven fluids
https://indico.ectstar.eu/event/230/contributions/speakers

- Foundations and Applications of Relativistic Hydrodynamics (Focus Week), Florence, Italy, 12-16 May 2025, Title: Electrically driven fluids https://www.ggi.infn.it/showevent.pl?id=518

- New Frontiers in Theoretical Physics - XXXVIII Convegno Nazionale di Fisica Teorica, Cortona, Italy, 20-23 May 2025, Title: Dissipative electrically driven fluids https://agenda.infn.it/event/44589/contributions/

Poster Presentations

- StatPhys29, Florence, Italy, 13-18 July 2025, Title: Dissipative electrically driven fluids

I attended the weekly seminars organised by INFN Genova throughout the whole year

Teaching Activity

- Tutor for the course "Fisica Generale 1"