# **Riccardo Grazi** – Second year Ph.D. report

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## Research Overview

During the second year of Ph.D. I continued my research on quantum batteries, which are quantum systems that store and transfer energy by exploiting only quantum features, and on the properties of spin chains in the thermodynamic limit. Building on the work I carried out during my first year, as well as on the knowledge I acquired, I extended the main project during the first half of my second year: to investigate how quantum phase transitions (i.e., changes in the ground-state properties of a system at zero temperature induced by varying a Hamiltonian parameter) influence the charging dynamics of quantum batteries. This line of work produced two new publications that analyze charging protocols across critical points and quantify the role of criticality in energy storage and extraction. More recently, I have focused on a phenomenon that has attracted growing attention: the quantum Mpemba effect. The quantum Mpemba effect denotes the counterintuitive situation in which two quantum systems prepared in different nonequilibrium states (often characterized by different effective temperatures or distances from a target steady state) relax toward the same stationary state at different rates, and, surprisingly, the system initially farther from equilibrium reaches the stationary state faster than the one that was initially closer. I am exploring whether the quantum Mpemba effect can be observed in battery setups and, importantly, whether it can be exploited to accelerate charging or discharging processes and thus constitute a genuine operational advantage for quantum energy-storage devices. In the following sections I describe in detail the research lines I carried forward this year, the methods employed, and the main results obtained.

1. In my first work of the second year, I investigated a broad class of quantum systems whose Hamiltonians can be mapped to $2\times2$ free fermion problems. Unlike my earlier studies, which considered only ground-state initial conditions, here I started from a thermal state and derived an analytical formula for the energy stored after a double sudden quantum quench of Hamiltonian parameters in arbitrary dimensions. I applied this result to several one-dimensional models (Ising, XY, cluster-Ising, and long-range SSH chains) and found that the stored energy retains a strong dependence on the quantum phase diagram of the charging Hamiltonian, with clear non-analytic signatures at critical points even at finite temperature and in the thermodynamic limit. Moreover, I showed that in Ising and XY chains one can achieve parameter-independent stored energy across wide regimes by appropriately choosing the initial state. Finally, by extending the analysis to cluster-Ising and SSH models, I demonstrated that this robustness persists even in the presence of long-range interactions.
2. The following work, which is currently under development, is a collaboration with Prof. Henrik Johannesson from the University of Göteborg (Sweden). We want to characterize a quantum battery based on a transverse-field Ising chain, evaluating its performances through the following figures of merit: stored energy, ergotropy (i.e. the maximum amount of work that can be extracted from a quantum state by means of unitary transformations) and power. The novelty of this study lies in two directions. First, we design a charging protocol in which the quenched parameter is varied over a finite time rather than abruptly. To understand the physics behind this new configuration I first performed an analytical calculation to derive the energy stored inside a single qubit and then I extended the analysis numerically to the many-body case of a spin chain, highlighting the differences between the two systems and how collective effects can increase the stability of the stored energy compared to the single-qubit case. Then, we extended our framework to include external noise, modeling the charging ramp as subject to Ornstein–Uhlenbeck correlations. By numerically solving the Lindblad master equation for the ensemble-averaged density matrix, we showed that ergotropy is very robust against small noises and this robustness is stronger if we design a ramp quench that crosses more than one quantum phase. Moreover, for intermediate noise strengths, we found a region in which ergotropy is even higher than the noiseless case, showing that environmental noise, under certain conditions, enhance battery performance rather than degrade it.

3. During the last part of my second year, I got more interested into the quantum Mpemba effect (QME). The project I'm currently pursuing consists in studying a spin chain initially prepared in a thermal state and presenting dissipation effects on each site. From the analytical side I derived the ergortopy for both the single and two-qubits scenario, while numerically I computed the ergotropies at different temperatures for a four-spins chain. Interestingly, the results reveal what can be interpreted as an inverse QME: there exists a time window during which the battery starting from the hottest initial state displays a higher ergotropy than the one starting from the coldest state. This is counterintuitive because, under standard thermodynamic intuition, higher initial temperatures generally reduce the amount of useful work extractable from a system; hence, one would expect the colder initial state to always yield the larger ergotropy. The observed inversion therefore highlights the nontrivial interplay of dissipation, coherence, and many-body effects in the dynamics of quantum batteries.

## Courses and exams

1. **Quantum Optics** - Exam given on 23/01/2025
2. **Energetics in the quantum regime** - Exam not given yet

## Schools and conferences

1. **Quantum Thermodynamics meets Quantum Transport** in Göteborg, Sweden (11/11/2024 - 15/11/2024). I presented a poster titled "Controlling energy storage crossing quantum phase transitions in an integrable spin quantum battery"
2. **International Conference on Quantum Energy (ICQE) 2025** in Padova, Italy (03/06/2025 - 06/06/2025). I presented a poster titled "Controlling energy storage crossing quantum phase transitions in an integrable spin quantum battery"
3. **Physics of Quantum Chips (PQC) 2025** in Gdansk, Poland (30/06/2025 - 04/07/2025). I presented a poster titled "Quantum batteries in the thermodynamic regime" and I did 3 hours of teaching activity about superconducting qubits.
4. **StatPhys29** in Florence, Italy (13/07/2025 - 18/07/2025). I presented a poster titled "Controlling energy storage crossing quantum phase transitions in an integrable spin quantum battery".
5. **Congresso Nazionale SIF** in Palermo, Italy (22/09/2025 - 25/09/2025). I will give a talk titled "Charging free fermion quantum batteries".
6. **ICE-10 Quantum Information in Spain** in Valencia, Spain (20/10/2025 - 24/10/2025). I will give a talk titled "Quantum batteries and quantum phase transitions".

## Publications

1. **R. Grazi**, D. Sacco Shaikh, M. Sassetti, N. Traverso Ziani and D. Ferraro, *Controlling Energy Storage Crossing Quantum Phase Transitions in an Integrable Spin Quantum Battery*, Physical Review Letters **133** (19), 197001 (2024). DOI: https://doi.org/10.1103/PhysRevLett.133.197001
2. **R. Grazi**, F. Cavaliere, N. Traverso Ziani and D. Ferraro, *Charging a Dimerized Quantum XY Chain*, Symmetry **2025**, 17(2), 220. DOI: https://doi.org/10.3390/sym17020220
3. **R. Grazi**, F. Cavaliere, M. Sassetti, D. Ferraro and N. Traverso Ziani, *Charging free fermion quantum batteries*, Chaos, Solitons & Fractals **196**, 116383 (2025). DOI: https://doi.org/10.1016/j.chaos.2025.116383

## Other Activities

1. Teaching assistant for the course "Fisica Generale 1" of the Bachelor's degree in Computer Engineering (30 hours of lectures).
2. Referee for *Physical Review A*.
3. Referee for *Physical Review E*.