Riccardo Grazi – First year Ph.D. report

Supervisors: Prof. Dario Ferraro, Dr. Niccolò Traverso Ziani, Prof. Fabio Cavaliere

Research Overview

During my first year of Ph.D., my research has focused on two main topics in theoretical condensed matter physics: quantum batteries, which are devices able to store and transfer energy exploiting purely quantum features, and spin chains. Since Alicki and Fannes introduced the concept of a quantum battery in 2013, research in this field has been developing rapidly. Studies are focused not only on identifying the best possible systems that can function as batteries but also on finding the most efficient charging protocols and methods to ensure stable energy storage for as long as possible. In this context, I have been studying the performance of spin chains as quantum batteries, specifically chains that can be analytically solved by mapping them onto free fermions, to understand if some of their properties can be used to gain advantages in terms of the energy stored by the device. Among the vast phenomenology characterizing these systems, I have primarily focused on studying quantum phase transitions, which are changes in the state of a quantum system that occur at zero temperature when a parameter in the system's Hamiltonian is varied. The goal was to better understand their role within the charging processes of a quantum battery and to determine whether improved performance could be observed in correspondence with a quantum phase transition in the system.

1. In my first work, which is currently under review for publication in Physical Review Letters, the system I studied was a dimerized anisotropic XY chain. After deriving the phase diagram, a charging protocol was defined based on a double quench of the dimerization parameter, unlike in existing works where the charger is typically an external system, which can be either classical or quantum, separate from the battery itself. By using a "stepwise" evolution of the dimerization parameter from an initial value δ_0 to a final value $\delta_0 + \delta_1$, the goal was to study the maximum energy stored by the battery per dimer as a function of δ_0, which, when varied, allows different regions of the previously determined phase diagram to be explored. This study revealed several key findings and led to the identification of three distinct working regimes. The first regime, which we refer to as the short-time regime, is characterized by a rapid charging process. Here, the optimal charging protocol involves evolving the system through a Hamiltonian that is factorized into disconnected dimers: this factorization allows for efficient energy transfer, maximizing the energy stored in the shortest amount of time. The second regime occurs at much longer timescales, specifically on the order of the system's recurrence time. In this long-time regime, the most favorable charging protocol still involves a fully dimerized Hamiltonian, but, as the system evolves, signatures of quantum phase transitions begin to emerge, indicating that the system is approaching critical points in the phase diagram. These transitions have a significant impact on the energy storage process, leading to potential enhancements in the stored energy. Finally, we explored a third regime, the thermodynamic one, in which the recurrance of the system is, from a temporal point of view, infinitely far from the peaks appearing at short times: in this scenario, the stored energy exhibits a remarkable stability, remaining consistent regardless of variations in charging time or other parameters. This stability with respect to the charging parameters is observed as long as the charging Hamiltonian and the battery Hamiltonian are separated by a single critical line in phase space. Eventually, to further validate these findings, we extended the analysis to a simpler model, the Ising model in a transverse field, which still exhibits a quantum phase transition in the thermodynamic limit despite being less complex, providing a useful comparison with the initially selected spin chain. The results confirmed that the quantum phase transition plays a crucial role in enhancing the energy storage capacity, even in this simpler case. This consistency across different models highlights the robustness of the observed effects and underscores the importance of quantum phase transitions in quantum battery systems.

2. In the second work, which is currently under development, we want to generalize the interesting effect discussed in the first paper to see if it is possible to extract a more general description of it and also to see if it remains valid when considering long-range interactions instead of the nearest-neighbor ones we used in the previous work. To do so, it is important to highlight the fact that the energy stored by the battery during the charging time strongly depends on the matrix $\mathcal{M}_k$ that relates the first set of fermionic operators, i.e., the one that characterizes the initial Hamiltonian where no parameter has been quenched yet, to the second set of fermionic operators, i.e., the one that describes the Hamiltonian we use for the temporal evolution and that depends on the quenched parameter. In this work, after finding the most general analytical form of this $\mathcal{M}_k$ matrix, we make a distinction between Hamiltonians that don't contain superconducting terms and Hamiltonians which do, presenting also a paradigmatic case for each scenario. Regarding the absence of superconducting terms, we choose the SSH model with first, second, and third-neighbor interactions: the main results we obtained show that the presence of second nearest-neighbor interactions doesn't impact the energy stored in the battery, making this scenario identical to the nearest-neighbor one, whereas, when adding third nearest-neighbor interactions, the results are different, but the effect is still present, making it a robust quantum many-body effect that also appears in spin chains characterized by long-range interactions. Finally, we also demonstrate the presence of this effect for Hamiltonians containing superconducting terms, such as the XY model and, for long-range interactions, the Cluster-Ising model.

Courses and exams

1. **Quantum Optics** - Exam not given yet
2. **Introduction to the Foundations of Quantum Mechanics and Applications** - Exam not given yet
3. **Machine Learning for Physics** (master course) - Exam not given yet

Schools and conferences

1. **Quantum Science Generation** from the 6th to the 10th of May 2024 in Trento, Italy. I presented a poster titled "Controlling energy storage crossing quantum phase transitions in an integrable spin quantum battery".
2. **Classical and Quantum Machine Learning for Condensed Matter Physics** from the 19th to the 21st of June 2024 held online.
3. **Non-equilibrium quantum many-body systems** from the 10th to the 12th of July 2024 in Genova, Italy. I gave a one-minute presentation talk.
4. **CMD 31** from the 2nd to the 6th of September 2024 in Braga, Portugal. I will give a talk titled "Controlling energy storage crossing quantum phase transitions in an integrable spin quantum battery".

Publications

1. **R. Grazi**, D. Sacco Shaikh, M. Sassetti, N. Traverso Ziani and D. Ferraro, *Controlling energy storage crossing quantum phase transitions in an integrable spin quantum battery*, Submitted in *Physical Review Letters*, arXiv:2402.09169 [quant-ph]

Other Activities

1. Referee for *Quantum Science and Technology*