PhD in Physics and Nanoscience XXXVI Cycle
Third Year Report

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Research Activity

Since the promizing results on a new covariant fracton model published at the beginning of this academic year [1], in the Third year of my PhD I focused on better understanding this theory and its physical consequences. From the gauge field theory perspective the study of the propagators of is quite peculiar. Indeed one can fix both a scalar gauge, as was done in [2], but also a vectorial one [3]. Such an ambiguity is related to the fact that the parameter in the gauge transformation defining fractons $\delta A_{\mu\nu} = \partial_\mu \partial_\nu \Lambda$ is a scalar function, for which the standard Faddeev-Popov procedure [4] would require a scalar gauge-fixing. At the same time this symmetry is a particular case of the diffeomorphism symmetry $\delta A_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$ defining Linearized Gravity (LG), which has a vectorial gauge parameter. Thus in this model, if we want for the limit of LG alone to exist, a vectorial gauge-fixing should be allowed. The uniqueness of this fracton theory is that both gauge-fixings are indeed possible without spoiling the number of degrees of freedom, which, as expected, should not depend on the gauge choice. Once this issue has been settled, and the fracton theory is well defined from a gauge field theory perspective, I looked into its physical implications. Firstly, in relation to my PhD thesis, I studied the consequences of adding a boundary to the full action, composed of two terms: $S_{tot} = g_1 S_{LG} + g_2 S_{fract}$, *i.e.* LG and pure fractons. There are many reasons for which this could lead to interesting results: the first is related to the defining property of fractons, *i.e.* limited mobility, for which these quasiparticles can move only in lower dimensional spaces (fractons are immobile, while other related quasiparticles can only move in one or two space dimensions), and for this reason they are often called "subdimensional particles" [5]. Thus identifying a lower dimensional surface, *i.e.* a boundary, comes quite natural. Another motivation is that fractons looks like a higher-rank Maxwell theory [6], and in my first paper [7] I showed that the electromagnetic theory has an induced physics on the boundary. Fractons could share also this similarity. Last but not least, some of these fracton models seem to be related to higher order topological insulators (HOTI) [8, 9], thus, as BF models with boundary are effective theories for the edge states of topological insulators [10], the addition of a boundary could be similarly related in the context of fractons and HOTI. The outcome, published in [11], is that indeed some relations with HOTI seem to exist, at least, from the point of view of the boundary algebra, which is a generalized Kac-Moody, in agreement with [9]. The 3D boundary theory of fractons is described in terms of two traceless-symmetric rank-2 tensors whose transformations coincide with the one that can be found in the so called "traceless scalar charge theory of fractons" [5, 6, 8] and share the same definition of traceless electric tensor field and Maxwell-like equations of motion. The most general 3D invariant action looks like a higher rank Maxwell-Chern-Simons theory [12], where the CS term plays the role of matter. However in this case it seems that there is no generation of "topological" mass, as it happens for the standard Maxwell-Chern-Simons theory. Additionally this boundary theory seem to have some relation with elasticity theory (which is a commonly known as "fracton-elasticity duality" [13]), and with self-dual massive gravity [14], which is currently being further investigated. The results obtained in this study are valid for both the full theory of fractons and LG ($g_1, g_2 \neq 0$), and for the pure fracton model ($g_1 = 0$), but not for the pure LG case. The reason is that, as explained at the beginning, LG is a theory defined by a more general symmetry (the diff symmetry), which plays an important role in the search for the induced boundary theory. Indeed it is the breaking of the symmetry due to the presence of the boundary that leads to the algebra and the degrees of freedom on the lower dimensional theory. If the symmetry changes, the boundary theory may change as well. For this reason I studied the pure LG case with boundary separately. As suspected, the results are radically different. Due to the nature of the symmetry, the algebra recovered on the boundary is a standard Kac-Moody algebra, which confirms the hypothesis on its existence, made in [15]. Moreover, the requirement of having a positive central charge constrains the sign of the coupling of the bulk theory, which in the LG case cannot be done through the computation of the energy-momentum tensor. The physical 3D theory is also different: we do not have traceless symmetric rank-2 tensors with a fractonic-like symmetry, but symmetric rank-2 tensors with a 3D diffeomorphism symmetry, which immediately lead to a theory of 3D LG on the boundary. All this being done, now I'm focussing on studying 3D fracton models as done for the 4D case in [1], also investigating the possibility of having a non-symmetric contribution in the action, which is mostly interesting from the point of view of the elasticity theory, thus of the fracton-elasticity duality.

Exams given

- Presentation of the Cargèse Summer School (see below)
- *From the Thermodynamics to the statistical Mechanics of Black Holes* - exam given
  Prof. S.Giusto
- *Theoretical Physics* - exam given
  Prof. G.Ridolfi

Schools/Workshops/Conferences

- Workshop: **Quantum Field Theory in Curved Spacetime II**
  Poster presentation: *Notes from the bulk: Metric dependence of the edge states of topological field theories.*
  24-26th May 2023, Granada, Spain
- Cargèse Series Summer School: **Advanced Summer School in Quantum Field Theory and Quantum Gravity**
  9-29th July 2023, Quy Nhon, Vietnam
- Nordita Workshop: **New perspectives on QFT with boundaries, impurities and defects**
  31st July-11th August 2023, Nordita, Stockholm, Sweden
- INFN Workshop: **New Frontiers in Theoretical Physics - XXXVII Convegno Nazionale di Fisica Teorica**
  Talk: *Maxwell theory of fractons.*
  27-29th September 2023, Cortona

Publications

- *Maxwell theory of fractons*
  E.Bertolini, N.Maggiore
  Phys.Rev.D **106** (2022), 125008, doi: 10.1103/PhysRevD.106.125008
- *Gauging fractons and linearized gravity*
  E.Bertolini, A.Blasi, A.Damonte, N.Maggiore
  Symmetry, **15** (2023), no.4, 945, doi: 10.3390/sym15040945
- *Covariant fracton gauge theory with boundary*
  E.Bertolini, N.Maggiore, G.Palumbo
  Phys.Rev.D **108** (2023), 025009, doi: 10.1103/PhysRevD.108.025009
- *Theory of a symmetric tensor field with boundary: Kac-Moody algebras in linearized gravity*
  E.Bertolini, N.Maggiore
  submitted for publication

Other activities

Teaching Assistant in Physics aimed at first-year chemical and electric engineers (100 hours).

References

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