# P.h.D. in Physics and Nanoscience XXXVI Cycle
# Second Year Report

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## Research Activity

During the second year of my PhD I continued the work on Quantum Field Theories (QFT) with (and without) boundary. Two are the main topics characterizing this second year:

- **BF** model in 3D with boundary, in curved spacetime ;
- **Fracton** models.

### The BF model

Following the results obtained by considering the Chern-Simons (CS) theory with boundary in a curved background [1], I studied the abelian BF model in 3 spacetime dimensions. As for CS, this is another example of a Topological Field Theory (TFT) [2], which is known to be related to the physics of Topological Insulators (TI) when a boundary is introduced [3]. The aim was to investigate if, as it happens for Hall systems [1], the velocities of the edge Degrees of Freedom (DoF) acquire a spacetime dependence through the induced metric. The standard bulk-to-boundary approach in flat spacetime provides only for constant edge velocities, while in the phenomenological framework local velocities are admitted by introducing an *ad hoc* potential in the action [4]. Can we say that when considering a boundary in a TFT, the curved spacetime paradigm is a formal way of allowing local velocities? We know that for CS that is true [1], but what about the 3D BF? This question is particularly relevant, in fact while in the edge states of fractional quantum Hall effect (which is the boundary description of CS) local (time dependent) velocities are observed [5], that is not the case for TI, and an affirmative answer would be a prediction. As I show in [6], the response is positive: local edge velocities arise also in the BF model as a consequence of the curved background and depend on the determinant of the induced metric. But there is more. Since two are the edge DoF and there are more parameters in the boundary conditions (which play a role in the edge velocities), the boundary physics is richer than the CS one. Depending on these parameters, the formal setup described in [6] allows for two edge velocities $v_+$ and $v_-$ which can be tuned in order to reproduce various cases :

- $\boldsymbol{v_+ v_- > 0}$: opposite velocities → chiral Luttinger liquids [7] ;
- $\boldsymbol{v_+ = v_-}$ : equal opposite velocities (Time Reversal) → Topological Insulators ;
- $\boldsymbol{v_+ v_- = 0}$: Left or Right mover not moving → Quantum Anomalous Hall Insulators [8] ;
- $\boldsymbol{v_+ v_- < 0}$: same direction, but $\mathcal{H} < 0$ → not allowed.

### Fracton models

Fractons are quasiparticles with the defining property of having restricted mobility [9]: "true" fractons cannot move at all, while other subdimensional particles can move in one or two space dimensions. The first observations of such behaviors have been in lattice models [10, 11, 12], but later fractons have been described as a rank-2 symmetric tensor gauge theory [13, 14]. This last case has many similarities with Maxwell theory (Hamiltonian, equations and so on), where "electric" and "magnetic" fields are tensors with two indices, and the restriction on mobility is generally achieved in the Literature by imposing by hand a Gauss-like constraint which leads to dipole conservation. Limitation on the possibility of displacement strongly suggests to identify some hypersurface on which the motion is/is not allowed. Additionally, as the standard Maxwell theory possess a boundary physics [15], this higher rank version might have its own, which can be related to higher order topological phases [9]. This was the aim of the second part of PhD year: to study fracton models in view of investigating its boundary physics, or find out if 4D fractons can be interpreted as the boundary physics of a higher dimensional theory. However, despite all the similarities with Maxwell theory, this higher rank version seems to be described by a non-covariant Lagrangian and if it were not enough, it is not homogeneous in the number of spatial derivatives [16]. These facts prevent the usual QFT approach, which requires, as first step, to build a covariant theory for fractons from which the results of the Literature could be reproduce. Therefore, starting from the covariant extension of the fracton symmetry, I built the most general invariant action. It is the sum of two independent terms, one of which corresponds to Linearized Gravity (LG). By defining a "field strength" $F_{\mu\nu\rho}$ invariant under the fracton symmetry (*i.e.* $\delta F = 0$), the whole action can be rewritten, and the non-LG term writes as $F^2$, exactly as for Maxwell. I identify this Maxwell-like contribution with fractons, in fact from its equations of motion, together with a Bianchi identity satisfied by $F_{\mu\nu\rho}$, the fractons' equations of the Literature [13] are recovered, with the correct "electric" and "magnetic" tensor fields. I also computed its energy-momentum tensor, and again, its components are the tensorial extension of its electromagnetic counterpart (energy density, Poynting and stress). I also analyzed the effect of introducing a non-constant $\theta$-term in the action [17], which is known to give modified Maxwell equations [18]. For fractons I found that the corresponding equations are the direct tensorial extension of the electromagnetic ones. Finally, by adding matter, a Lorentz-like force arise as a term of the conservation of the energy-momentum tensor. This point is particularly relevant, in fact we recovered, without asking any additional requirement, the expression for the Lorentz force postulated "by intuition" (*sic*) in [13]. The physical interpretation is clear: while fractons do not move, dipoles do, according to the dynamics induced by the tensor electric and magnetic fields previously found. This work opens at many possibilities: studying the boundary physics, but also the links with LG, a massive extension, and a higher dimensional version at $d = 6$, which appears to be the most natural for this fracton model. These just to cite a few.

## Exams given

- Presentation of the ICTP Summer School (see below) - soon.

## Schools/Workshops/Conferences

- INFN Workshop: **Theories of Fundamental Interactions (TFI)**
  Poster presentation: *Topological BF description of 2D accelerated chiral edge modes.*
  13-15th June 2022, Venice
- ICTP School: **Summer School on Cosmology 2022**
  4-15th July 2022, Trieste
- Workshop: **Avenues of Quantum Field Theory in Curved Spacetime**
  Talk: *Notes from the bulk: metric dependence of the edge states of topological field theories.*
  14-16th September 2022, Genoa

## Publications

- *Notes from the bulk: metric dependence of the edge states of Chern-Simons theory*
  E.Bertolini, G.Gambuti, N.Maggiore, Phys.Rev.D **104** (2021) 10, 105011, doi: 10.1103/PhysRevD.104.105011
- *Topological BF Description of 2D Accelerated Chiral Edge Modes*
  E.Bertolini, F.Fecit, N.Maggiore, Symmetry, **14** (2022), no.4, 675, doi: 10.3390/sym14040675
- *Maxwell theory of fractons*
  E.Bertolini, N.Maggiore, submitted for publication

## Other activities

Teaching Assistant in Physics aimed at first-year chemical and electric engineers (30 hours).

## References


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